Preparing for A level Study Support Pack - Summer 2022



Subject: Further Mathematics

The aim of this pack is to help you bridge the gap between GCSE and A level. It is specific to one of the many A level subjects that are taught at The Bedford Sixth Form and we encourage you to work through all the relevant packs for the subjects that you would like to study.

Things to Consider

Welcome mathematicians! Hopefully you find yourself reading this because despite current circumstances you are expecting some of the best results from your GCSE mathematics qualifications. The biggest point to have in mind before choosing Further Mathematics as one of your options, is that you will also have to choose Mathematics as well. This means that up to two thirds of your study time will be committed to mathematics. Therefore, we would strongly recommend that it be a subject that you are not only good at, but you also find a great deal of satisfaction from solving problems and are happy with spending a lot of time developing your mathematical skills.

The subject specific entry requirements for the course is a grade 8 for a reason, instead of a grade 7 for our normal mathematics A-Level. The course is a lot more mathematically demanding and challenging; however, this is what makes your success in the subject that much more rewarding. So, if you are up for a challenge and cannot wait to get started with the course then this study support pack is for you. We look forward to seeing you come September.

What is included in this study support pack?

The key topics and skills that you should be confident with are very much the same as with normal mathematics, so make sure that you start with that study support pack first before starting this one. Haven't said that, to make the transition at the start of the course as smooth as possible, you need to be even more prepared on these topics than your normal mathematics peers.

So, with that in mind, we've pulled together all of the transition materials provided by Edexcel covering all of the necessary key topics for core pure mathematics. Each topic has some practice questions before introducing more challenging extension questions, and there are a lot of them. While working through these, you should take the opportunity to consider whether this is something you enjoy doing and can see yourself doing for the next two years.

Similar to normal mathematics, should you want more resources to practice and prepare with, then try these websites:

- https://mathsmadeeasy.co.uk/gcse-maths-revision/ (Level 8-9)
- https://examqa.com/gcse/maths/ (Grade 8/9)

There are many other online resources that I am sure you will have heard of and used over the past year or more. The main goal is to ensure that you are as confident with all of the topics covered in this study support pack as you can be, so if you prefer using another website or resource then by all means use it.

Also included in this pack is a copy of last year's UKMT Senior Kangaroo, which, as mentioned in the Mathematics study support pack, is a follow-on round for those who are achieve at least a gold in the Senior Mathematical Challenge. It is a 60-minute competition consisting of 20 problems.

Unlike the Senior Mathematical Challenge, the answer to each Senior Kangaroo problem is a positive integer less than 1000. Achieving a high score requires engagement with the problems without the benefit of five options to choose from. One of last year's first year Further Mathematics students made it to this round. Could it be you next year?

The <u>minimum expectation</u> is that you <u>complete at least 100 points</u> worth of questions, but take it as another opportunity to problem solve and challenge yourself. Try to solve any of the problems that you can and write up your solutions. If this is something that you do not enjoy, are you sure that you will enjoy the challenge that comes with studying A-Level Further Mathematics?

More papers can be found here: <u>https://www.ukmt.org.uk/competitions/solo/senior-kangaroo/archive</u>

Essential and Recommended Equipment

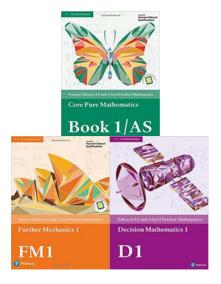
As well as ensuring that you're caught up with and are confident on all of the key topics covered in this study support pack. Another thing that you can do to prepare for and hit the ground running when you start the A-Level Further Mathematics course is to buy some essential equipment beforehand. That way it will be ready for when you start with us and the focus can be keeping on top of and practicing the material covered in classes.

The only additional pieces of essential equipment, beyond those covered in the Mathematics study support pack, are copies of your own Further Mathematics textbooks. We use these three textbooks:

- Edexcel AS and A level Further Mathematics Core Pure Mathematics Book 1/AS

- Edexcel AS and A level Further Mathematics Further Mechanics 1

- Edexcel AS and A level Further Mathematics Decision Mathematics 1



Should you be thinking of taking maths or a science qualification beyond A-Levels, a graphing calculator might be a good investment. This is not essential, but should you want to buy one instead of the standard one mentioned in the Mathematics study support pack, then we'd recommend the Casio fx-CG50 Colour Graphic Calculator.

Contact the Department

Do you need any further help or guidance on preparing for the A-Level Further Mathematics course? Then feel free to contact the department by email. There is currently only one teacher within the maths department that teaches the course. Here are his details: Kris Li - <u>kli@bedfordsixthform.ac.uk</u>

Practice (10 points)

1	Expa	and.			Watch out!
	a	3(2x-1)	b	$-2(5pq + 4q^2)$	
	c	$-(3xy-2y^2)$			When multiplying (or
					dividing) positive and negative numbers, if
2	-	and and simplify.			the signs are the same
		7(3x+5)+6(2x-8)		8(5p-2) - 3(4p+9)	the answer is '+'; if the
	c	9(3s+1)-5(6s-10)	d	2(4x-3) - (3x+5)	signs are different the
					answer is '-'.
3	Expa	and.		L	
		3x(4x+8)		$4k(5k^2-12)$	
	c	$-2h(6h^2+11h-5)$	d	$-3s(4s^2-7s+2)$	
4	-	and and simplify.			
		$3(y^2 - 8) - 4(y^2 - 5)$		2x(x+5)+3x(x-7)	
	c	4p(2p-1) - 3p(5p-2)	d	3b(4b-3) - b(6b-9)	
5	Exp	and $\frac{1}{2}(2y-8)$			
6	Expa	and and simplify.			
	a	13 - 2(m + 7)	b	$5p(p^2+6p)-9p(2p-3)$	
7	The	diagram shows a rectangle.			
		e down an expression, in terms of.	x, foi	the area of	
		ectangle.		3x-5	
		w that the area of the rectangle can	be w	vritten as	
	$21x^{-}$	-35x			7x
8	Eve	and and simplify.			
0	-	(x+4)(x+5)	b	(x+7)(x+3)	
		(x + 7)(x - 2)		(x + 7)(x + 5) (x + 5)(x - 5)	
		(x + 7)(x - 2) (2x + 3)(x - 1)		(x+3)(x-3) (3x-2)(2x+1)	
		(2x+3)(x-1) (5x-3)(2x-5)		(3x-2)(2x+1) (3x-2)(7+4x)	
	-	(3x - 3)(2x - 3) (3x + 4y)(5y + 6x)		(3x-2)(7+4x) $(x+5)^2$	
		$(3x + 4y)(3y + 6x)^{2}$		$(x + 3)^2$ $(4x - 3y)^2$	
	ĸ	(2x + 1)	1	(4x 3y)	
Ex	ten	d (5 points)			
9	Expa	and and simplify $(x + 3)^2 + (x - 4)^2$			
10	Expa	and and simplify.			
		(1)(2)		$(1)^{2}$	
	a	$\left(x+\frac{1}{x}\right)\left(x-\frac{2}{x}\right)$	b	$\left(x+\frac{1}{x}\right)^2$	



Practice (10 points)

1 Simplify.

	1 9		
a	$\sqrt{45}$	b	$\sqrt{125}$
c	$\sqrt{48}$	d	$\sqrt{175}$
e	$\sqrt{300}$	f	$\sqrt{28}$
g	$\sqrt{72}$	h	$\sqrt{162}$

Hint
One of the two
numbers you
choose at the start
must be a square
number.

7.

Watch out! **b** $\sqrt{45} - 2\sqrt{5}$ $\sqrt{72} + \sqrt{162}$ a Check you have c $\sqrt{50} - \sqrt{8}$ d $\sqrt{75} - \sqrt{48}$ chosen the highest square number at **f** $2\sqrt{12} - \sqrt{12} + \sqrt{27}$ e $2\sqrt{28} + \sqrt{28}$ the start.

3 Expand and simplify.

a	$(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})$	b	$(3+\sqrt{3})(5-\sqrt{12})$
c	$(4-\sqrt{5})(\sqrt{45}+2)$	d	$(5+\sqrt{2})(6-\sqrt{8})$

4 Rationalise and simplify, if possible.

 $\frac{1}{\sqrt{5}}$ $\mathbf{b} = \frac{1}{\sqrt{11}}$ a d $\frac{2}{\sqrt{8}}$ c $\frac{2}{\sqrt{7}}$ f $\frac{5}{\sqrt{5}}$ e $\frac{2}{\sqrt{2}}$ h $\frac{\sqrt{5}}{\sqrt{45}}$ $\mathbf{g} = \frac{\sqrt{8}}{\sqrt{24}}$

a
$$\frac{1}{3-\sqrt{5}}$$
 b $\frac{2}{4+\sqrt{3}}$ **c** $\frac{6}{5-\sqrt{2}}$



Extend (5 points)

- 6 Expand and simplify $(\sqrt{x} + \sqrt{y})(\sqrt{x} \sqrt{y})$
- 7 Rationalise and simplify, if possible.

a
$$\frac{1}{\sqrt{9}-\sqrt{8}}$$
 b $\frac{1}{\sqrt{x}-\sqrt{y}}$



Practice (10 points)

1	Evaluate. a 14 ⁰	b	3 ⁰	c	5 ⁰	d	x^0
2	Evaluate. a $49^{\frac{1}{2}}$	b	$64^{\frac{1}{3}}$	c	$125^{\frac{1}{3}}$	d	$16^{\frac{1}{4}}$
3	Evaluate. a $25^{\frac{3}{2}}$	b	$8^{\frac{5}{3}}$	c	$49^{\frac{3}{2}}$	d	$16^{\frac{3}{4}}$
4	Evaluate. a 5^{-2}	b	4 ⁻³	c	2-5	d	6 ⁻²
5	Simplify. a $\frac{3x^2 \times x^3}{2x^2}$ c $\frac{3x \times 2x^3}{2x^3}$ e $\frac{y^2}{y^{\frac{1}{2}} \times y}$ g $\frac{(2x^2)^3}{4x^0}$	b d f h	$\frac{10x^5}{2x^2 \times x}$ $\frac{7x^3y^2}{14x^5y}$ $\frac{c^{\frac{1}{2}}}{c^2 \times c^{\frac{3}{2}}}$ $\frac{x^{\frac{1}{2}} \times x^{\frac{3}{2}}}{x^{-2} \times x^3}$		Watch out! Remember that any value raises the power of z is 1. This is the rule $a^0 = 1$.	ed to ero	
6	Evaluate. a $4^{-\frac{1}{2}}$ d $16^{\frac{1}{4}} \times 2^{-3}$	b e	$27^{-\frac{2}{3}}$ $\left(\frac{9}{16}\right)^{-\frac{1}{2}}$	c f	$9^{-\frac{1}{2}} \times 2^{3}$ $\left(\frac{27}{64}\right)^{-\frac{2}{3}}$		
7	Write the following as	a single	power of <i>x</i> .		_		

a $\frac{1}{x}$ **b** $\frac{1}{x^7}$ **c** $\sqrt[4]{x}$ **d** $\sqrt[5]{x^2}$ **e** $\frac{1}{\sqrt[3]{x}}$ **f** $\frac{1}{\sqrt[3]{x^2}}$



9

8 Write the following without negative or fractional powers.

a	x^{-3}	b x^0	с	$x^{\frac{1}{5}}$
d	$x^{\frac{2}{5}}$	e $x^{-\frac{1}{2}}$	f	$x^{-\frac{3}{4}}$

Write the following in the form ax^n .						
a $5\sqrt{x}$	b $\frac{2}{x^3}$	c	$\frac{1}{3x^4}$			
d $\frac{2}{\sqrt{x}}$	e $\frac{4}{\sqrt[3]{x}}$	f	3			

Extend (5 points)

10 Write as sums of powers of *x*.

a
$$\frac{x^5 + 1}{x^2}$$
 b $x^2\left(x + \frac{1}{x}\right)$ **c** $x^{-4}\left(x^2 + \frac{1}{x^3}\right)$



Practice (10 points)

1	Fac	ctorise.		
	a	$6x^4y^3 - 10x^3y^4$	b	$21a^3b^5 + 35a^5b^2$
	c	$25x^2y^2 - 10x^3y^2 + 15x^2y^3$		
2	Fac	ctorise		
	a	$x^2 + 7x + 12$	b	$x^2 + 5x - 14$
	c	$x^2 - 11x + 30$	d	$x^2 - 5x - 24$
	e	$x^2 - 7x - 18$	f	$x^2 + x - 20$
	g	$x^2 - 3x - 40$	h	$x^2 + 3x - 28$
3	Fac	ctorise		
	a	$36x^2 - 49y^2$	b	$4x^2 - 81y^2$
	c	$18a^2 - 200b^2c^2$		

Hint

Take the highest common factor outside the bracket.

4 Factorise

a	$2x^2 + x - 3$	b	$6x^2 + 17x + 5$
c	$2x^2 + 7x + 3$	d	$9x^2 - 15x + 4$
e	$10x^2 + 21x + 9$	f	$12x^2 - 38x + 20$

5 Simplify the algebraic fractions.

a	$\frac{2x^2 + 4x}{x^2 - x}$	b	$\frac{x^2+3x}{x^2+2x-3}$
c	$\frac{x^2-2x-8}{x^2-4x}$	d	$\frac{x^2 - 5x}{x^2 - 25}$
e	$\frac{x^2 - x - 12}{x^2 - 4x}$	f	$\frac{2x^2 + 14x}{2x^2 + 4x - 70}$

6 Simplify

a
$$\frac{9x^2 - 16}{3x^2 + 17x - 28}$$

b $\frac{2x^2 - 7x - 15}{3x^2 - 17x + 10}$
c $\frac{4 - 25x^2}{10x^2 - 11x - 6}$
d $\frac{6x^2 - x - 1}{2x^2 + 7x - 4}$

Extend (5 points)

7 Simplify $\sqrt{x^2 + 10x + 25}$

8 Simplify
$$\frac{(x+2)^2 + 3(x+2)^2}{x^2 - 4}$$



Practice (10 points)

1 Write the following quadratic expressions in the form $(x + p)^2 + q$

a	$x^2 + 4x + 3$	b	$x^2 - 10x - 3$
c	$x^2 - 8x$	d	$x^{2} + 6x$
e	$x^2 - 2x + 7$	f	$x^{2} + 3x - 2$

2 Write the following quadratic expressions in the form $p(x + q)^2 + r$ a $2x^2 - 8x - 16$ b $4x^2 - 8x - 16$ c $3x^2 + 12x - 9$ d $2x^2 + 6x - 8$

3 Complete the square.

a	$2x^2 + 3x + 6$	b	$3x^2 - 2x$
c	$5x^2 + 3x$	d	$3x^2 + 5x + 3$

Extend (5 points)

4 Write $(25x^2 + 30x + 12)$ in the form $(ax + b)^2 + c$.



Practice (10 points)

Solve by factorisation.
Solve by factorisation.

a	$6x^2 + 4x = 0$	b	$28x^2 - 21x = 0$
c	$x^2 + 7x + 10 = 0$	d	$x^2 - 5x + 6 = 0$
e	$x^2 - 3x - 4 = 0$	f	$x^2 + 3x - 10 = 0$
g	$x^2 - 10x + 24 = 0$	h	$x^2 - 36 = 0$
i	$x^2 + 3x - 28 = 0$	j	$x^2 - 6x + 9 = 0$
k	$2x^2 - 7x - 4 = 0$	1	$3x^2 - 13x - 10 = 0$

2 Solve by factorisation.

4

a	$x^2 - 3x = 10$	b	$x^2 - 3 = 2x$
c	$x^2 + 5x = 24$	d	$x^2 - 42 = x$
e	x(x+2) = 2x + 25	f	$x^2 - 30 = 3x - 2$
g	$x(3x+1) = x^2 + 15$	h	3x(x-1) = 2(x+1)



Get all terms onto one side of the equation.

3 Solve by completing the square.

a	$x^2 - 4x - 3 = 0$	b	$x^2 - 10x + 4 = 0$
c	$x^2 + 8x - 5 = 0$	d	$x^2 - 2x - 6 = 0$
e	$2x^2 + 8x - 5 = 0$	f	$5x^2 + 3x - 4 = 0$

Sol	ve by completing the square.	Hint		
a	(x-4)(x+2) = 5	IIIIt		
b	$2x^2 + 6x - 7 = 0$	Get all terms		
c	$x^2 - 5x + 3 = 0$	onto one side of the equation.		

5 Solve by using the quadratic formula, giving your solutions in surd form.

a $3x^2 + 6x + 2 = 0$ **b** $2x^2 - 4x - 7 = 0$

- 6 Solve the equation $x^2 7x + 2 = 0$ by using the quadratic formula. Give your solutions in the form $\frac{a \pm \sqrt{b}}{c}$, where *a*, *b* and *c* are integers.
- 7 Solve $10x^2 + 3x + 3 = 5$ by using the quadratic formula. Give your solution in surd form.

Hint

Get all terms onto one side of the equation.



Extend (5 points)

- 8 Choose an appropriate method to solve each quadratic equation, giving your answer in surd form when necessary.
 - **a** 4x(x-1) = 3x-2
 - **b** $10 = (x+1)^2$
 - **c** x(3x-1) = 10



Practice (10 points)

- 1 Sketch the graph of $y = -x^2$.
- 2 Sketch each graph, labelling where the curve crosses the axes. **a** y = (x+2)(x-1) **b** y = x(x-3) **c** y = (x+1)(x+5)
- 3 Sketch each graph, labelling where the curve crosses the axes. a $y = x^2 - x - 6$ b $y = x^2 - 5x + 4$ c $y = x^2 - 4$ d $y = x^2 + 4x$ e $y = 9 - x^2$ f $y = x^2 + 2x - 3$
- 4 Sketch the graph of $y = 2x^2 + 5x 3$, labelling where the curve crosses the axes.

Extend (5 points)

- 5 Sketch each graph. Label where the curve crosses the axes and write down the coordinates of the turning point.
 - **a** $y = x^2 5x + 6$ **b** $y = -x^2 + 7x 12$ **c** $y = -x^2 + 4x$
- 6 Sketch the graph of $y = x^2 + 2x + 1$. Label where the curve crosses the axes and write down the equation of the line of symmetry.



Practice (10 points)

Solve these simultaneous equations using the elimination method.

- 1
 4x + y = 8 2
 3x + y = 7

 x + y = 5 3x + 2y = 5

 3
 4x + y = 3 4x + y = 7

 3x y = 11 x 4y = 5
- **5** 2x + y = 11x - 3y = 9**6** 2x + 3y = 113x + 2y = 4

Solve these simultaneous equations using the substitution method.

y = x - 48 y = 2x - 32x + 5y = 435x - 3y = 119 2y = 4x + 510 2x = y - 29x + 5y = 228x - 5y = -1111 3x + 4y = 812 3y = 4x - 72y = 3x - 42x - y = -1314 3x + 2y + 1 = 013 3x = y - 12y - 2x = 34y = 8 - x

Extend (5 points)

15 Solve the simultaneous equations 3x + 5y - 20 = 0 and $2(x + y) = \frac{3(y - x)}{4}$.



Practice (10 points)

Solve these simultaneous equations.

- **2** y = 6 x1 y = 2x + 1 $x^2 + y^2 = 10$ $x^2 + y^2 = 20$ **3** y = x - 34 y = 9 - 2x $x^2 + y^2 = 5$ $x^2 + y^2 = 17$ **5** y = 3x - 5 $6 \quad y = x - 5$ $y = x^2 - 2x + 1$ $y = x^2 - 5x - 12$ y = x + 58 y = 2x - 1 $x^2 + y^2 = 25$ $x^2 + xy = 24$
- 9 y = 2x $y^2 - xy = 8$ 10 2x + y = 11xy = 15

Extend (5 points)

11 x - y = 1 $x^2 + y^2 = 3$ **12** y - x = 2 $x^2 + xy = 3$



Practice (10 points)

- 1 Solve these pairs of simultaneous equations graphically.
 - **a** y = 3x 1 and y = x + 3
 - **b** y = x 5 and y = 7 5x
 - **c** y = 3x + 4 and y = 2 x
- 2 Solve these pairs of simultaneous equations graphically.
 - **a** x + y = 0 and y = 2x + 6
 - **b** 4x + 2y = 3 and y = 3x 1
 - **c** 2x + y + 4 = 0 and 2y = 3x 1
- 3 Solve these pairs of simultaneous equations graphically.
 - **a** y = x 1 and $y = x^2 4x + 3$
 - **b** y = 1 3x and $y = x^2 3x 3$
 - c y = 3 x and $y = x^2 + 2x + 5$
- 4 Solve the simultaneous equations x + y = 1 and $x^2 + y^2 = 25$ graphically.

Extend (5 points)

- 5 a Solve the simultaneous equations 2x + y = 3 and $x^2 + y = 4$
 - i graphically
 - ii algebraically to 2 decimal places.
 - **b** Which method gives the more accurate solutions? Explain your answer.

Hint

Rearrange the equation to make *y* the subject.



Practice (10 points)

1	Sol	ve these inequalities.				
	a	4x > 16	b	$5x-7 \leq 3$	c	$1 \ge 3x + 4$
	d	5 - 2x < 12	e	$\frac{x}{2} \ge 5$	f	$8 < 3 - \frac{x}{3}$
2	Sol	ve these inequalities.				
	a	$\frac{x}{5} < -4$	b	$10 \ge 2x + 3$	c	7 - 3x > -5
3	Sol	ve				
	a	$2 - 4x \ge 18$	b	$3 \le 7x + 10 < 45$	c	$6-2x \ge 4$
	d	4x + 17 < 2 - x	e	4-5x<-3x	f	$-4x \ge 24$
4	Sol	ve these inequalities.				
		3t + 1 < t + 6		b $2(3n-1)$	$) \ge n +$	5
5	Sol	VA				
3					a / =	`
	a	3(2-x) > 2(4-x) +	4	b $5(4-x)$	> 3(5 -	(-x) + 2

Extend (5 points)

6 Find the set of values of x for which 2x + 1 > 11 and 4x - 2 > 16 - 2x.



Practice (10 points)

- 1 Find the set of values of x for which $(x + 7)(x 4) \le 0$
- 2 Find the set of values of x for which $x^2 4x 12 \ge 0$
- **3** Find the set of values of x for which $2x^2 7x + 3 < 0$
- 4 Find the set of values of x for which $4x^2 + 4x 3 > 0$
- 5 Find the set of values of x for which $12 + x x^2 \ge 0$

Extend (5 points)

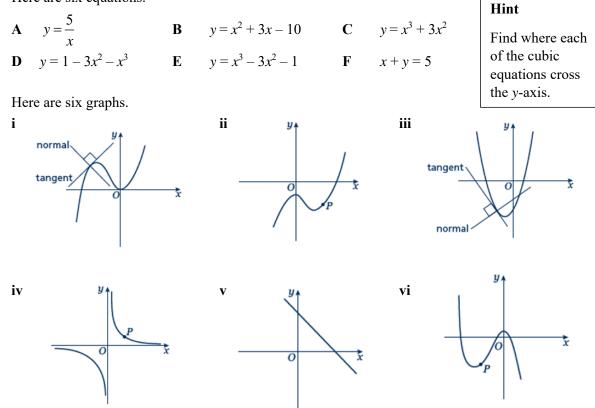
Find the set of values which satisfy the following inequalities.

- $\mathbf{6} \qquad x^2 + x \le \mathbf{6}$
- 7 x(2x-9) < -10
- 8 $6x^2 \ge 15 + x$



Practice (10 points)

1 Here are six equations.



- **a** Match each graph to its equation.
- **b** Copy the graphs ii, iv and vi and draw the tangent and normal each at point P.

Sketch the following graphs

- **2** $y = 2x^3$ **3** y = x(x-2)(x+2)
- 4 y = (x + 1)(x + 4)(x 3)
- 6 $y = (x-3)^2(x+1)$

8

y =

$$\frac{3}{x}$$
 Hint: Look at the shape of $y = \frac{a}{x}$ 9 $y = -\frac{2}{x}$
in the second key point.

Extend (5 points)

10 Sketch the graph of
$$y = \frac{1}{x+2}$$
 11 Sketch the graph of $y = \frac{1}{x-1}$

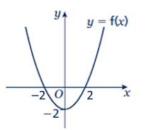


5 y = (x+1)(x-2)(1-x)

7 $y = (x-1)^2(x-2)$

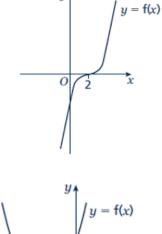
Practice (10 points)

1 The graph shows the function y = f(x). Copy the graph and on the same axes sketch and label the graphs of y = f(x) + 4 and y = f(x + 2).



y,

2 The graph shows the function y = f(x). Copy the graph and on the same axes sketch and label the graphs of y = f(x + 3) and y = f(x) - 3.



20

x

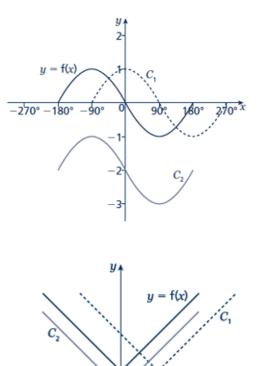
3 The graph shows the function y = f(x). Copy the graph and on the same axes sketch the graph of y = f(x - 5).



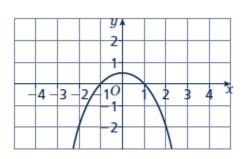
4 The graph shows the function y = f(x) and two transformations of y = f(x), labelled C_1 and C_2 . Write down the equations of the translated curves C_1 and C_2 in function form.

5 The graph shows the function y = f(x) and two transformations of y = f(x), labelled C_1 and C_2 . Write down the equations of the translated curves C_1 and C_2 in function form.

- **6** The graph shows the function y = f(x).
 - **a** Sketch the graph of y = f(x) + 2
 - **b** Sketch the graph of y = f(x + 2)

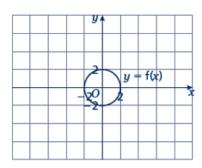


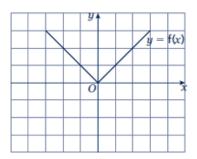
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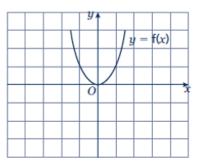


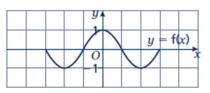


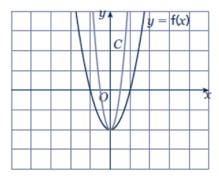
- 7 The graph shows the function y = f(x).
 - **a** Copy the graph and on the same axes sketch and label the graph of y = 3f(x).
 - **b** Make another copy of the graph and on the same axes sketch and label the graph of y = f(2x).
- 8 The graph shows the function y = f(x). Copy the graph and on the same axes sketch and label the graphs of y = -2f(x) and y = f(3x).
- 9 The graph shows the function y = f(x). Copy the graph and, on the same axes, sketch and label the graphs of y = -f(x) and $y = f(\frac{1}{2}x)$.
- 10 The graph shows the function y = f(x). Copy the graph and, on the same axes, sketch the graph of y = -f(2x).
- 11 The graph shows the function y = f(x) and a transformation, labelled *C*. Write down the equation of the translated curve *C* in function form.





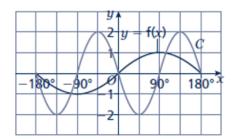




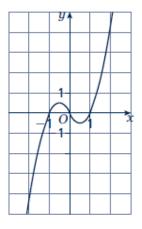




12 The graph shows the function y = f(x) and a transformation labelled *C*. Write down the equation of the translated curve *C* in function form.



- **13** The graph shows the function y = f(x).
 - **a** Sketch the graph of y = -f(x).
 - **b** Sketch the graph of y = 2f(x).



Extend (5 points)

- **14** a Sketch and label the graph of y = f(x), where f(x) = (x 1)(x + 1).
 - **b** On the same axes, sketch and label the graphs of y = f(x) 2 and y = f(x + 2).
- 15 a Sketch and label the graph of y = f(x), where f(x) = -(x + 1)(x 2).
 - **b** On the same axes, sketch and label the graph of $y = f(-\frac{1}{2}x)$.



Practice (10 points)

1 Find the gradient and the *y*-intercept of the following equations.

a	y = 3x + 5	b	$y = -\frac{1}{2}x - 7$	
c	2y = 4x - 3	d	x + y = 5	Hint Rearrange the equations
e	2x - 3y - 7 = 0	f	5x + y - 4 = 0	Rearrange the equations to the form $y = mx + c$

2 Copy and complete the table, giving the equation of the line in the form y = mx + c.

Gradient	y-intercept	Equation of the line
5	0	
-3	2	
4	-7	

3 Find, in the form ax + by + c = 0 where a, b and c are integers, an equation for each of the lines with the following gradients and y-intercepts.

a	gradient $-\frac{1}{2}$, y-intercept -7	b	gradient 2, y-intercept 0
c	gradient $\frac{2}{3}$, y-intercept 4	d	gradient -1.2, y-intercept -2

- 4 Write an equation for the line which passes though the point (2, 5) and has gradient 4.
- 5 Write an equation for the line which passes through the point (6, 3) and has gradient $-\frac{2}{3}$
- 6 Write an equation for the line passing through each of the following pairs of points.

a	(4, 5), (10, 17)	b	(0, 6), (-4, 8)
c	(-1, -7), (5, 23)	d	(3, 10), (4, 7)

Extend (5 points)

7 The equation of a line is 2y + 3x - 6 = 0. Write as much information as possible about this line.



Practice (10 points)

- Find the equation of the line parallel to each of the given lines and which passes through each of 1 the given points.
 - **b** y = 3 2x (1, 3) **a** y = 3x + 1 (3, 2) 2x + 4y + 3 = 0 (6, -3) **d** 2y - 3x + 2 = 0 (8, 20)
 - c

Hint Find the equation of the line perpendicular to $y = \frac{1}{2}x - 3$ which 2 If $m = \frac{a}{b}$ then the negative passes through the point (-5, 3). reciprocal $-\frac{1}{m} = -\frac{b}{a}$

- Find the equation of the line perpendicular to each of the given lines and which passes through 3 each of the given points.
 - **b** $y = -\frac{1}{3}x + \frac{1}{2}$ (2, 13) **a** y = 2x - 6 (4, 0) **c** x - 4y - 4 = 0 (5, 15) **d** 5y + 2x - 5 = 0 (6, 7)
- In each case find an equation for the line passing through the origin which is also perpendicular 4 to the line joining the two points given.
 - a (4, 3), (-2, -9)(0, 3), (-10, 8)b

Extend (5 points)

5 Work out whether these pairs of lines are parallel, perpendicular or neither.

	y = 2x + 3 $y = 2x - 7$		y = 3x $2x + y - 3 = 0$		y = 4x - 3 $4y + x = 2$
d	3x - y + 5 = 0 $x + 3y = 1$	e	2x + 5y - 1 = 0 $y = 2x + 7$	f	2x - y = 6 $6x - 3y + 3 = 0$

- The straight line L_1 passes through the points A and B with coordinates (-4, 4) and (2, 1), 6 respectively.
 - Find the equation of L₁ in the form ax + by + c = 0a

The line L_2 is parallel to the line L_1 and passes through the point C with coordinates (-8, 3). Find the equation of L₂ in the form ax + by + c = 0b

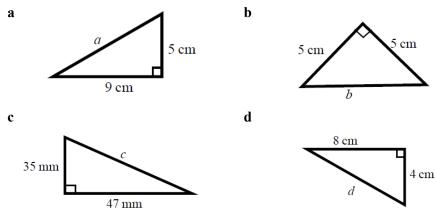
The line L_3 is perpendicular to the line L_1 and passes through the origin.

Find an equation of L_3 с

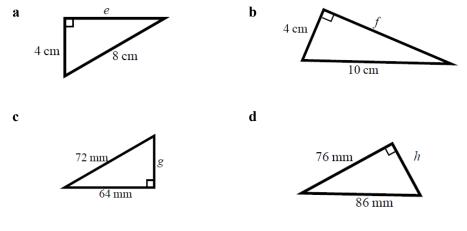


Practice (10 points)

1 Work out the length of the unknown side in each triangle. Give your answers correct to 3 significant figures.

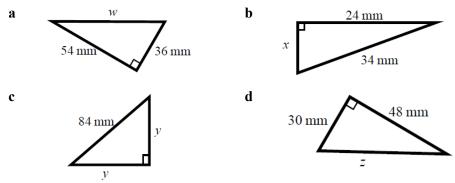


2 Work out the length of the unknown side in each triangle. Give your answers in surd form.

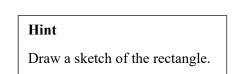




3 Work out the length of the unknown side in each triangle. Give your answers in surd form.

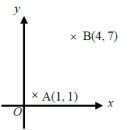


4 A rectangle has length 84 mm and width 45 mm. Calculate the length of the diagonal of the rectangle. Give your answer correct to 3 significant figures.

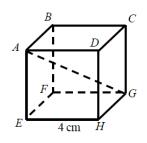


Extend (5 points)

- 5 A yacht is 40 km due North of a lighthouse. A rescue boat is 50 km due East of the same lighthouse. Work out the distance between the yacht and the rescue boat. Give your answer correct to 3 significant figures.
- 6 Points A and B are shown on the diagram. Work out the length of the line AB. Give your answer in surd form.



7 A cube has length 4 cm.Work out the length of the diagonal AG.Give your answer in surd form.



Hint

Draw a diagram using the information given in the question.



Practice (10 points)

- Paul gets paid an hourly rate. The amount of pay (£*P*) is directly proportional to the number of hours (*h*) he works. When he works 8 hours he is paid £56. If Paul works for 11 hours, how much is he paid?
- 2 x is directly proportional to y. x = 35 when y = 5.
 - **a** Find a formula for *x* in terms of *y*.
 - **b** Sketch the graph of the formula.
 - c Find x when y = 13.
 - **d** Find y when x = 63.
- 3 *Q* is directly proportional to the square of *Z*. Q = 48 when Z = 4.
 - **a** Find a formula for Q in terms of Z.
 - **b** Sketch the graph of the formula.
 - c Find Q when Z = 5.
 - **d** Find Z when Q = 300.
- 4 y is directly proportional to the square of x. x = 2 when y = 10.
 - **a** Find a formula for *y* in terms of *x*.
 - **b** Sketch the graph of the formula.
 - c Find x when y = 90.
- 5 *B* is directly proportional to the square root of *C*. C = 25 when B = 10.
 - **a** Find *B* when C = 64.
 - **b** Find C when B = 20.
- 6 C is directly proportional to D. C = 100 when D = 150. Find C when D = 450.
- 7 y is directly proportional to x. x = 27 when y = 9. Find x when y = 3.7.
- 8 *m* is proportional to the cube of *n*. m = 54 when n = 3. Find *n* when m = 250.

Hint

Substitute the values given for *P* and *h* into the formula to calculate *k*.



Extend (5 points)

- 9 *s* is inversely proportional to *t*.
 - **a** Given that s = 2 when t = 2, find a formula for s in terms of t.
 - **b** Sketch the graph of the formula.
 - **c** Find *t* when s = 1.
- 10 *a* is inversely proportional to *b*. a = 5 when b = 20.
 - **a** Find *a* when b = 50.
 - **b** Find *b* when a = 10.
- 11 *v* is inversely proportional to *w*.
 - w = 4 when v = 20.
 - **a** Find a formula for *v* in terms of *w*.
 - **b** Sketch the graph of the formula.
 - **c** Find w when v = 2.
- 12 L is inversely proportional to W. L = 12 when W = 3. Find W when L = 6.
- 13 *s* is inversely proportional to *t*. s = 6 when t = 12.
 - **a** Find s when t = 3.
 - **b** Find t when s = 18.
- 14 y is inversely proportional to x^2 . y = 4 when x = 2. Find y when x = 4.
- 15 y is inversely proportional to the square root of x. x = 25 when y = 1. Find x when y = 5.
- 16 *a* is inversely proportional to *b*. a = 0.05 when b = 4.
 - **a** Find *a* when b = 2.
 - **b** Find *b* when a = 2.



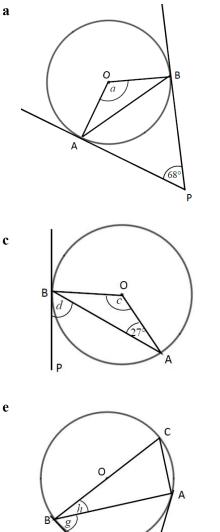
Practice (10 points)

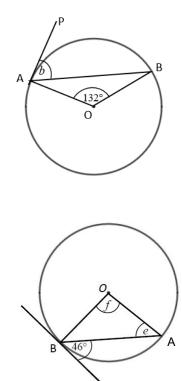
Work out the size of each angle marked with a letter. 1 Give reasons for your answers.

b

d

b

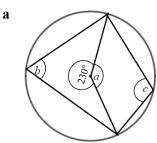


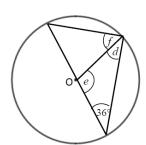


P

2

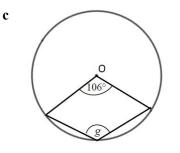
Work out the size of each angle marked with a letter. Give reasons for your answers.





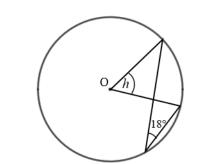


d



Hint

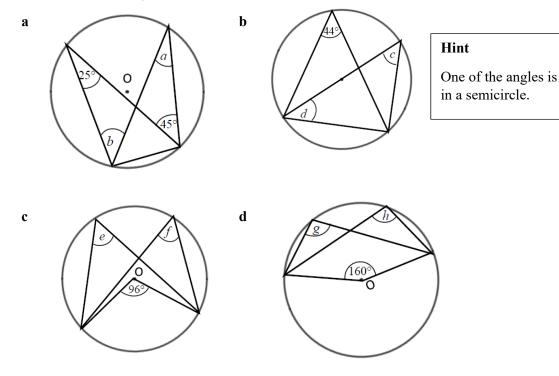
The reflex angle at point O and angle g are subtended by the same arc. So the reflex angle is twice the size of angle g.



Hint

Angle 18° and angle *h* are subtended by the same arc.

3 Work out the size of each angle marked with a letter. Give reasons for your answers.

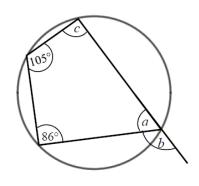




a

b

4 Work out the size of each angle marked with a letter. Give reasons for your answers.

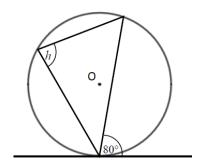


Hint

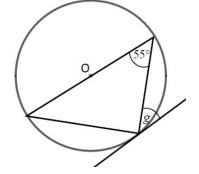
с

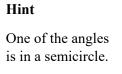
An exterior angle of a cyclic quadrilateral is equal to the opposite interior angle.

1 1 88³



d





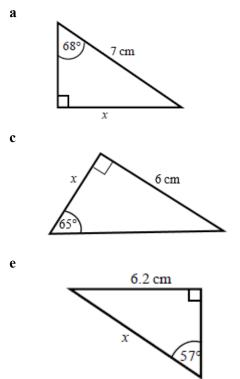
Extend (5 points)

5 Prove the alternate segment theorem.

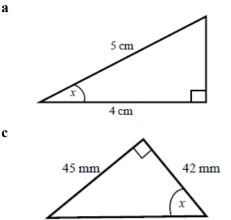


Practice (10 points)

1 Calculate the length of the unknown side in each triangle. Give your answers correct to 3 significant figures.



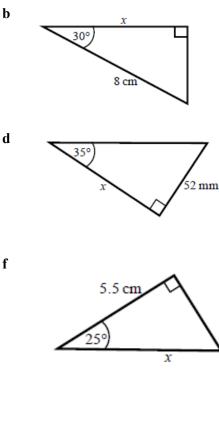
2 Calculate the size of angle *x* in each triangle. Give your answers correct to 1 decimal place.

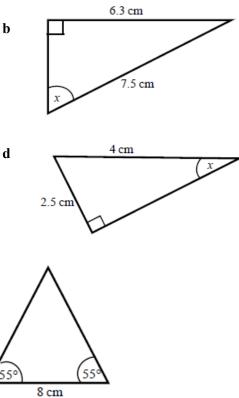


3 Work out the height of the isosceles triangle. Give your answer correct to 3 significant figures.

Hint:

Split the triangle into two right-angled triangles.





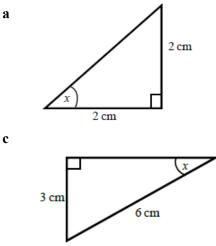


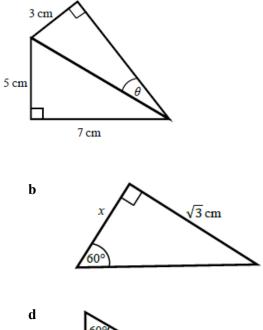
Calculate the size of angle θ . 4 Give your answer correct to 1 decimal place.

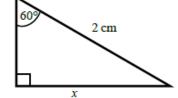
Hint:

First work out the length of the common side to both triangles, leaving your answer in surd form.

Find the exact value of *x* in each triangle. 5

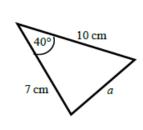






Work out the length of the unknown side in each triangle. 6 Give your answers correct to 3 significant figures.

55 mm



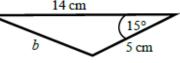
95





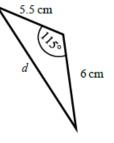
a

с



d

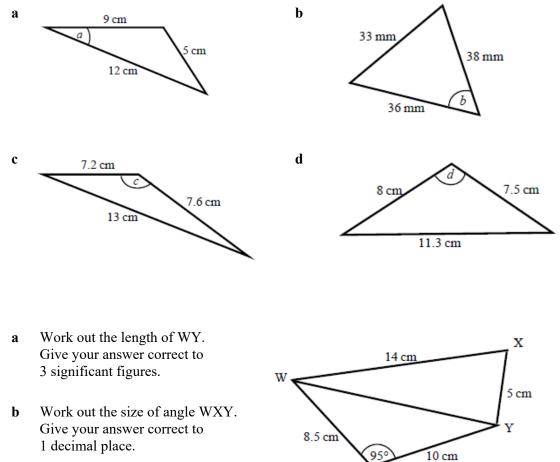
b





8

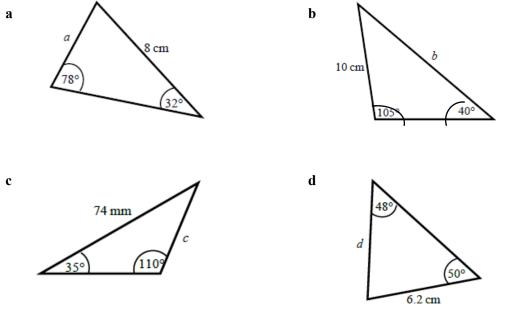
7 Calculate the angles labelled θ in each triangle. Give your answer correct to 1 decimal place.



Ż

9 Find the length of the unknown side in each triangle.

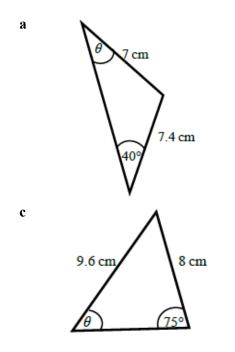
Give your answers correct to 3 significant figures.





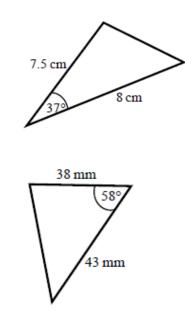


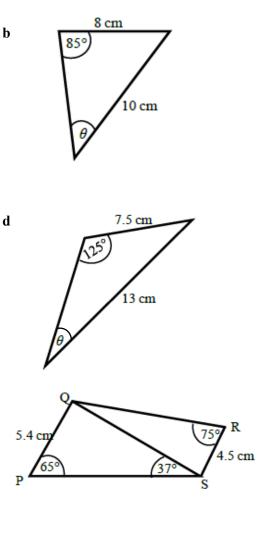
10 Calculate the angles labelled θ in each triangle. Give your answer correct to 1 decimal place.



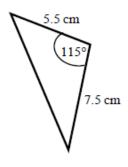
- 11 a Work out the length of QS. Give your answer correct to 3 significant figures.
 - **b** Work out the size of angle RQS. Give your answer correct to 1 decimal place.
- Work out the area of each triangle.Give your answers correct to 3 significant figures.
 - a

c





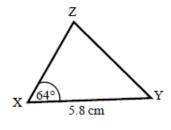
b





13 The area of triangle XYZ is 13.3 cm².Work out the length of XZ.

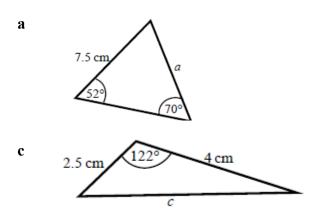
Hint:

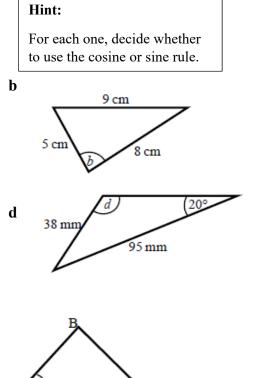


Rearrange the formula to make a side the subject.

Extend (5 points)

14 Find the size of each lettered angle or side. Give your answers correct to 3 significant figures.





630

16.5 cm

С

15 The area of triangle ABC is 86.7 cm².Work out the length of BC.Give your answer correct to 3 significant figures.



Practice (10 points)

Change the subject of each formula to the letter given in the brackets.

- 1 $C = \pi d \quad [d]$ 2 $P = 2l + 2w \quad [w]$ 3 $D = \frac{S}{T} \quad [T]$ 4 $p = \frac{q - r}{t} \quad [t]$ 5 $u = at - \frac{1}{2}t \quad [t]$ 6 $V = ax + 4x \quad [x]$ 7 $\frac{y - 7x}{2} = \frac{7 - 2y}{3} \quad [y]$ 8 $x = \frac{2a - 1}{3 - a} \quad [a]$ 9 $x = \frac{b - c}{d} \quad [d]$ 7 2x + 3
- **10** $h = \frac{7g 9}{2 + g}$ [g] **11** e(9 + x) = 2e + 1 [e] **12** $y = \frac{2x + 3}{4 x}$ [x]
- 13 Make *r* the subject of the following formulae.

a
$$A = \pi r^2$$
 b $V = \frac{4}{3}\pi r^3$ **c** $P = \pi r + 2r$ **d** $V = \frac{2}{3}\pi r^2 h$

14 Make *x* the subject of the following formulae.

$$\mathbf{a} \quad \frac{xy}{z} = \frac{ab}{cd} \qquad \qquad \mathbf{b} \quad \frac{4\pi cx}{d} = \frac{3z}{py^2}$$

15 Make sin *B* the subject of the formula $\frac{a}{\sin A} = \frac{b}{\sin B}$

16 Make $\cos B$ the subject of the formula $b^2 = a^2 + c^2 - 2ac \cos B$.

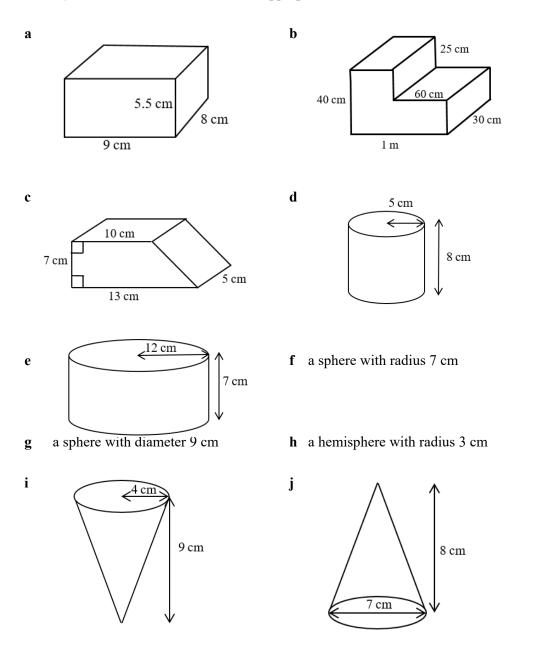
Extend (5 points)

- 17 Make x the subject of the following equations.
 - **a** $\frac{p}{q}(sx+t) = x-1$ **b** $\frac{p}{q}(ax+2y) = \frac{3p}{q^2}(x-y)$

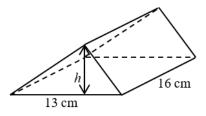


Practice (10 points)

1 Work out the volume of each solid. Leave your answers in terms of π where appropriate.



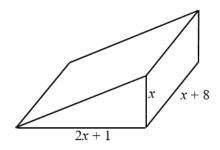
- 2 A cuboid has width 9.5 cm, height 8 cm and volume 1292 cm³. Work out its length.
- 3 The triangular prism has volume 1768 cm³. Work out its height.

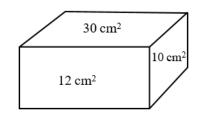




Extend (5 points)

- The diagram shows a solid triangular prism. All the measurements are in centimetres. The volume of the prism is V cm³. Find a formula for V in terms of x. Give your answer in simplified form.
- 5 The diagram shows the area of each of three faces of a cuboid.The length of each edge of the cuboid is a whole number of centimetres.Work out the volume of the cuboid.





6 The diagram shows a large catering size tin of beans in the shape of a cylinder.

The tin has a radius of 8 cm and a height of 15 cm. A company wants to make a new size of tin. The new tin will have a radius of 6.7 cm. It will have the same volume as the large tin. Calculate the height of the new tin. Give your answer correct to one decimal place.

7 The diagram shows a sphere and a solid cylinder. The sphere has radius 8 cm.

The solid cylinder has a base radius of 4 cm and a height of h cm.

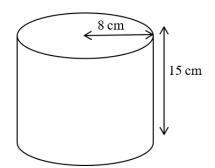
The total surface area of the cylinder is half the total surface area of the sphere.

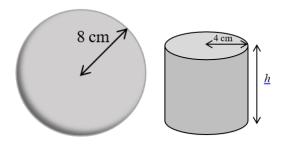
Work out the ratio of the volume of the sphere to the volume of the cylinder.

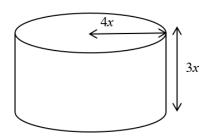
Give your answer in its simplest form.

8 The diagram shows a solid metal cylinder. The cylinder has base radius 4x and height 3x. The cylinder is melted down and made into a sphere of radius *r*.

Find an expression for r in terms of x.









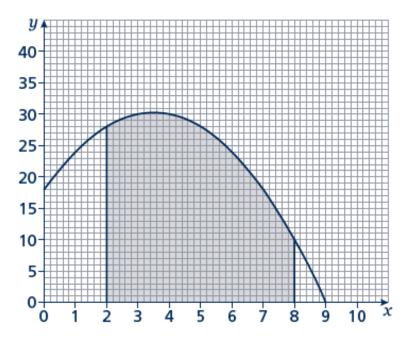
Practice (10 points)

1 Estimate the area of the region between the curve y = (5 - x)(x + 2) and the *x*-axis from x = 1 to x = 5. Use four strips of width 1 unit.

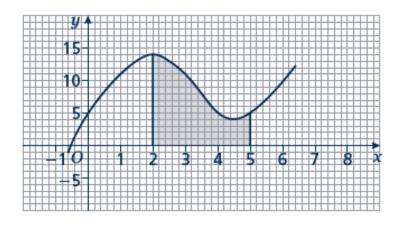
Hint:

For a full answer, remember to include 'units²'.

Estimate the shaded area shown on the axes.Use six strips of width 1 unit.



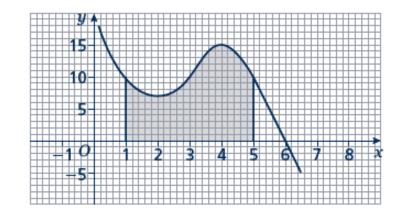
- 3 Estimate the area of the region between the curve $y = x^2 8x + 18$ and the x-axis from x = 2 to x = 6. Use four strips of width 1 unit.
- 4 Estimate the shaded area. Use six strips of width $\frac{1}{2}$ unit.



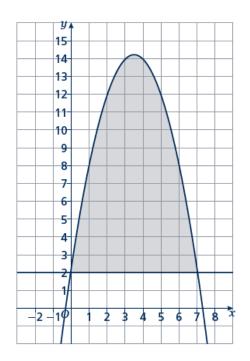




- 5 Estimate the area of the region between the curve $y = -x^2 4x + 5$ and the *x*-axis from x = -5 to x = 1. Use six strips of width 1 unit.
- 6 Estimate the shaded area. Use four strips of equal width.



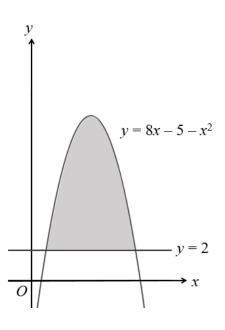
- 7 Estimate the area of the region between the curve $y = -x^2 + 2x + 15$ and the *x*-axis from x = 2 to x = 5. Use six strips of equal width.
- 8 Estimate the shaded area. Use seven strips of equal width.



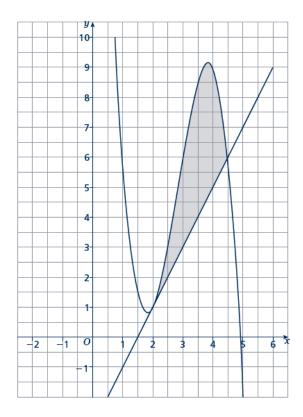


Extend (5 points)

9 The curve $y = 8x - 5 - x^2$ and the line y = 2 are shown in the sketch. Estimate the shaded area using six strips of equal width.



10 Estimate the shaded area using five strips of equal width.





Challenge (5 points per correct solution)







Friday 29 November 2019

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a member of the Association Kangourou sans Frontières



England & Wales: Year 13 or below Scotland: S6 or below Northern Ireland: Year 14 or below

INSTRUCTIONS

- 1. Do not open the paper until the invigilator tells you to do so.
- 2. Time allowed: **60 minutes**. No answers, or personal details, may be entered after the allowed time is over.
- 3. The use of blank or lined paper for rough working is allowed; squared paper, calculators and measuring instruments are forbidden.
- 4. Use a B or an HB non-propelling pencil to record your answer to each problem as a three-digit number from 000 to 999.

Pay close attention to the example on the Answer Sheet that shows how to code your answers.

5. **Do not expect to finish the whole paper in the time allowed.** The questions in this paper have been arranged in approximate order of difficulty with the harder questions towards the end. You are not expected to complete all the questions during the time. You should bear this in mind when deciding which questions to tackle.

6. Scoring rules:

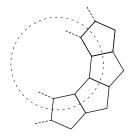
5 marks are awarded for each correct answer; There is no penalty for giving an incorrect answer.

7. The questions on this paper are designed to challenge you to think, not to guess. You will gain more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. This paper is about solving interesting problems, not about lucky guessing.

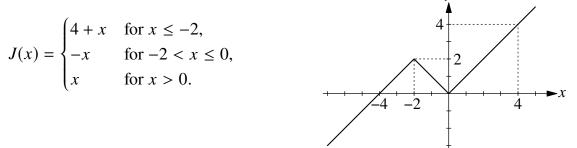
Enquiries about the Senior Kangaroo should be sent to:

UK Mathematics Trust, School of Mathematics, University of Leeds, Leeds LS2 9JT**a** 0113 343 2339enquiry@ukmt.org.ukwww.ukmt.org.uk

- **1.** What is the sum of all the factors of 144?
- **2.** When I noticed that $2^4 = 4^2$, I tried to find other pairs of numbers with this property. Trying 2 and 16, I realised that 2^{16} is larger than 16^2 . How many times larger is 2^{16} ?
- **3.** The two diagonals of a quadrilateral are perpendicular. The lengths of the diagonals are 14 and 30. What is the area of the quadrilateral?
- 4. The integer *n* satisfies the inequality $n + (n + 1) + (n + 2) + \dots + (n + 20) > 2019$. What is the minimum possible value of *n*?
- 5. Identical regular pentagons are arranged in a ring. The partially completed ring is shown in the diagram. Each of the regular pentagons has a perimeter of 65. The regular polygon formed as the inner boundary of the ring has a perimeter of P. What is the value of P?



- 6. For natural numbers *a* and *b* we are given that $2019 = a^2 b^2$. It is known that a < 1000. What is the value of *a*?
- 7. How many positive? integers n exist such that both $\frac{n+1}{3}$ and 3n + 1 are three-digit integers?
- **8.** The function J(x) is defined by:



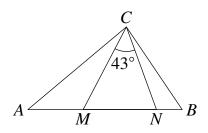
How many distinct real solutions has the equation J(J(J(x))) = 0?

- 9. What is the smallest three-digit number *K* which can be written as $K = a^b + b^a$, where both *a* and *b* are one-digit positive integers?
- 10. What is the value of $\sqrt{13 + \sqrt{28 + \sqrt{281}}} \times \sqrt{13 \sqrt{28 + \sqrt{281}}} \times \sqrt{141 + \sqrt{281}}$?

11. In the triangle *ABC* the points *M* and *N* lie on the side *AB* such that AN = AC and BM = BC.

We know that $\angle MCN = 43^{\circ}$.

Find the size in degrees of $\angle ACB$.



12. What is the value of $A^2 + B^3 + C^5$, given that:

$$A = \sqrt[3]{16\sqrt{2}}$$
$$B = \sqrt{9\sqrt[3]{9}}$$
$$C = \left[\left(\sqrt[5]{2}\right)^2\right]^2$$

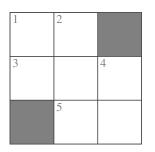
- 13. The real numbers *a* and *b*, where a > b, are solutions to the equation $3^{2x} 10 \times 3^{x+1} + 81 = 0$. What is the value of $20a^2 + 18b^2$?
- 14. A number N is the product of three distinct primes. How many distinct factors does N^5 have?
- 15. Five Bunchkins sit in a horizontal field. No three of the Bunchkins are sitting in a straight line. Each Bunchkin knows the four distances between her and each of the others. Each Bunchkin calculates and then announces the total of these distances. These totals are 17, 43, 56, 66 and 76. A straight line is painted joining each pair of Bunchkins. What is the total length of paint required?
- **16.** The real numbers *x* and *y* satisfy the equations:

xy - x = 180 and y + xy = 208. Let the two solutions be (x_1, y_1) and (x_2, y_2) .

What is the value of $x_1 + 10y_1 + x_2 + 10y_2$?

- **17.** In triangle *ABC*, $\angle BAC$ is 120°. The length of *AB* is 123. The point *M* is the midpoint of side *BC*. The line segments *AB* and *AM* are perpendicular. What is the length of side *AC*?
- 18. An integer is said to be *chunky* if it consists only of non-zero digits by which it is divisible when written in base 10.For example, the number 936 is Chunky since it is divisible by 9, 3 and 6.How many chunky integers are there between 13 and 113?

- 19. The square ABCD has sides of length 105. The point M is the midpoint of side BC. The point N is the midpoint of BM. The lines BD and AM meet at the point P. The lines BD and AN meet at the point Q. What is the area of triangle *APQ*?
- 20. Each square in this cross-number can be filled with a non-zero digit such that all of the conditions in the clues are fulfilled. The digits used are not necessarily distinct. What is the answer to 3 ACROSS?



ACROSS

- 1. A composite factor of 1001
- 3. Not a palindrome

DOWN

- 1. One more than a prime, one less than a prime
- 2. A multiple of 9
- 5. pq^3 where p, q prime and $p \neq q = 4$. p^3q using the same p, q as 5 ACROSS